Revised: May 29, 2012

Electron Structure and Inversion*)

DAN PETRU DĂNESCU e-mail:dpdanescu@yahoo.com

Abstract

Having summarily dealt with plane geometric inversion in the special case of transforming a line into a circle, enforcing the tangential condition requires dealing with the kinematics application of this transformation and its implications over interpreting elementary electric charge, Compton wavelength and electron structure. There is supposed to be close connection between this kinematics application of geometric inversion and time reversal, space inversion and charge conjugation

Keywords: Geometric inversion, electron, electron structure, fundamental physical constants**.**

1.Introduction

The inversion (from the Latin verb *invertere*, which means "to reverse") was first defined by Apollonius (3rd century BC). Significant contributions to its study were brought by J. Plucker (1831) and G. Bellavitis (1837). W. Thomson (1843) applied to solve electrostatic problems.

We owe the "inversion" name to A. Bravais (1850).

The fact that the geometrical inversion can be presented as a process which transforms an infinite line into a circle with a diameter no matter how small, in other words it makes a connection between the big infinity and the small infinity, suggests us the possibility of linking the cosmic space-time with elementary particles, namely the electron.

This geometrical transformation, correlated with the fact that the world must be seen as an ever evolving process, where "everything flows" « *pantha rhei*, idea owed to Heraclit (6th century BC) and in modern times to A.N. Whitehead (1933) , [1] », opens the perspective of a kinematical view of nature, different than the classical Hamiltonian method, whivh offers us a new point of view.

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*) English version of the article: " Transformarea prin inversiune. Aplicatii " (Transformation by inversion. Applications), Gazeta Matematica, Anul LXXXVIII, nr.3 (1978).

2.The plane geometric inversion

In the metric space, suppose a point O, named pole, and a number K named *inversion power* [2-7].

This number can have positive values $(K = \rho^2)$ or negative $(K = -\rho^2)$. We assume $K = \rho^2$, because otherwise, beside the inversion we have to take into consideration an O centre symmetry.

The inversion $I_{O,K}$ *is the punctual transformation which associates to each point M an image M' (Fig.1) so that on the OM radius the relation*

$$OM \cdot OM' = K . \tag{1}$$

should be satisfied.

M is the inverse of M'.

We observe immediately that:

-any point has an inverse, except the pole whose inverse is at infinite;

-if the point M' is the inverse of M, then, reciprocally, this last one is the inverse of the first one.

It is called inverse figure of a figure F, the figure F' formed from the inverse of the points that form the figure F.

As we are going to show, it takes interest the inverse figure of *a* line that does not go through the pole of inversion (the line is evidently invariant if goes through the pole).

Be O the inversion pole, K the inversion power and a the line whose inverse figure we are looking for (Fig.2a). We consider a certain point M on the a line and M' its inverse. We have to determine the multitude of M' points. First we will consider a particular position of the M point and that is the point A, the foot of the perpendicular from O to a an and than a certain position of the point M on the line a. Be M' and A' inverse for the points M and A. This means that the relations

$$OM' \cdot OM = K , \tag{2}$$

$$OA' \cdot OA = K . \tag{3}$$

are satisfied.

Comparing the relations (2) and (3) we have:

$$\frac{OM}{OA'} = \frac{OA}{OM'} \quad . \tag{4}$$

This equality leads to the conclusion that the triangles OMA and $OM'A'$ are alike, because they have in common the angle $\sphericalangle MOA$ between the sides are respectively proportional. From the fact that the two triangles are alike results that the other angles are respectively equal too, so

$$\sphericalangle OA'M' = \sphericalangle OMA \quad ; \quad \sphericalangle OM'A' = \sphericalangle OAM.$$

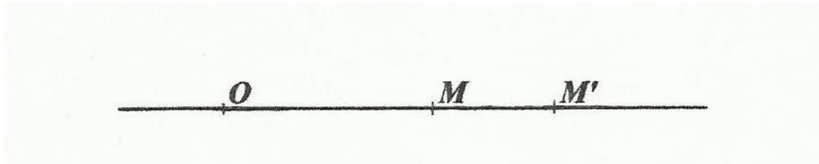


Fig.1 Inversion on a line

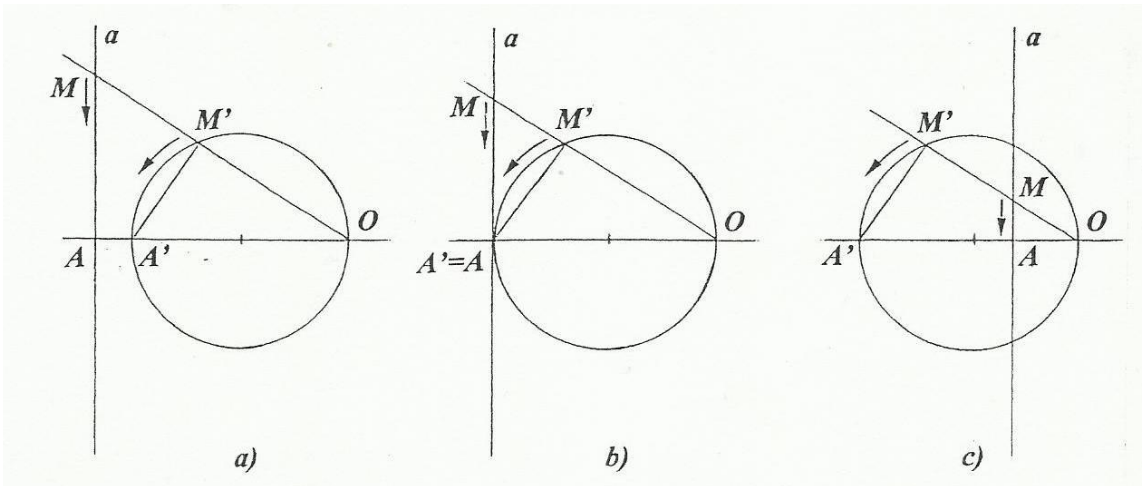


Fig.2 The three cases of transformation by inversion of a line in a circle

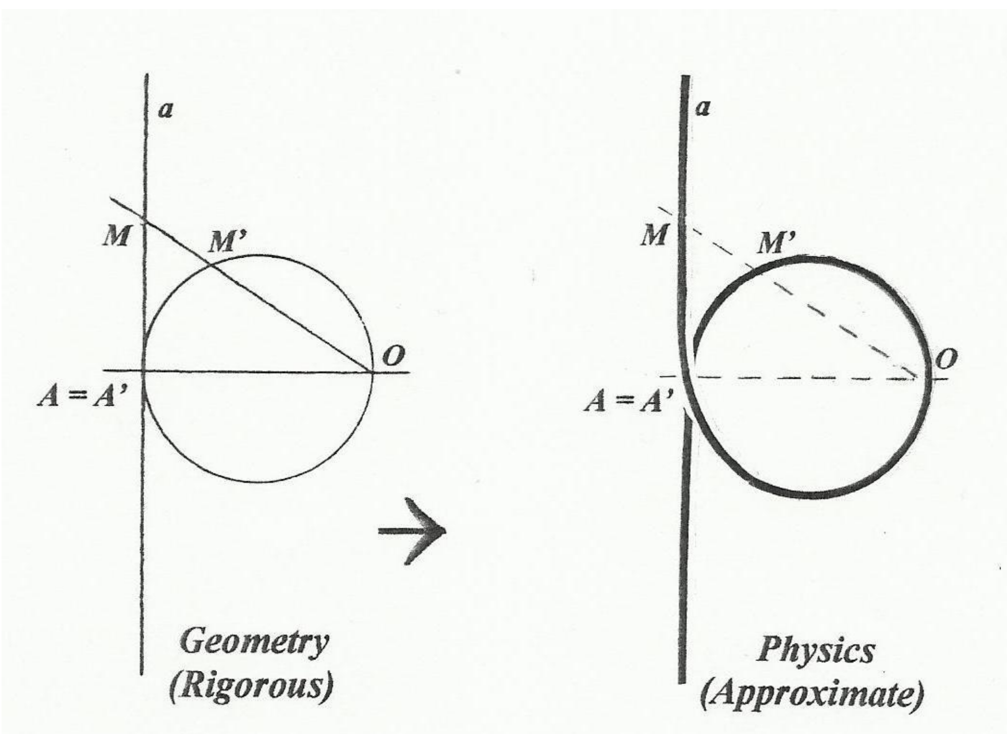


Fig.3 Transition from the geometric inversion on a line in a circle to the physics inversion on a elastic filament in a loop. Equation ***OA.OA' = OM.OM' = K*** is valid in both cases.

The inverse of the point A is fix and the segment OA' is constant. Because $\sphericalangle OAM$ is of 90 degrees, it results that his equal, $\sphericalangle OM'A'$ is of 90 degrees. From this it is deducted that when the point M moves on the line a, M' the inverse of M describe a circle with the diameter OA'. Also, any point of the circle with the OA' diameter is the transforming by inversion of a point on the line a.

Because from (3) we hawe,

$$OA' = \frac{K}{OA}, \tag{5}$$

it results that there are three cases:

I). $K < OA^2$, so

$$OA' = \frac{K}{OA} < OA, \tag{6}$$

the circle notintersecting the line a (Fig. 2a).

II). $K = OA^2$, so

$$OA' = \frac{K}{OA} = OA, \tag{7}$$

the circle being tangent to the line a (Fig. 2b).

III). $K > OA^2$, resulting

$$OA' = \frac{K}{OA} > OA, \tag{8}$$

so, the circle intersects the line a (Fig. 2c).

3. Applications of kinematic's electron

We will consider on two applications of the transformations by inversion related to what have been enlarged until now. Of a big importance in these applications is the choice of the pole and the determination of the inversion power. As we will see, only the physics reasoning will lead us to their choosing and determination.

Transition from the geometric inversion to physical inversion is shown in Fig.3.

Be a overhand knot as the one from Fig.4a, built on an elastic fiber, unextensible and of negligible section, identifyed of fundamental field line, which can move by slipping along the fiber with the speed v. We propose to find the equation of the movement resorting to the transformation by inversion.

We deal with this problem considering it in a plane (due to the negligible section of the fiber) but we have not to forget that in fact the knot represents a spatial construction. The form of the knot is - by analogy - identical with the one resulted from a similar knot made on a tensioned steal wire. We observe that at these knots the two chords substretch arcs of about 60^o, so they have radius length.

During a time interval *(Δt)*, the fiber length $\Delta\ell$ (the segment *AG*) will be transformed by inversion evidently in a circle (Fig. 4a). In order to bring the problem in the case II, we have to calculate the length of the segment *AH* (perpendicular on the diameter *OA*)

$$AH \approx AG \cos 30^o \approx \frac{\sqrt{3}}{2} \Delta\ell \ . \tag{9}$$

The diameter of the circle resulted by inversion will be

$$OA \approx \frac{\sqrt{3}}{2\pi} \Delta\ell \ . \tag{10}$$

Choosing a natural system of measurement units in which the smallest segment from Fig. 4a is taken as measurement unit *(AP ≈ 1 cm.)*, results *OA ≈ 2 cm.,* and $K \approx 2^2$ cm^2. In this case, the inversion equation (3) becomes:

$$OA \ . \ \frac{\sqrt{3}}{2\pi} \Delta\ell \ \approx 2^2 \ cm^2. \tag{11}$$

Dividing the equality (11) by the time unit *(Δt = 1s.)*, we get:

$$OA \ . \ \frac{\sqrt{3}}{2\pi} \frac{\Delta\ell}{\Delta t} \approx 2^2 \ cm^2/s \ . \tag{12}$$

Having in mind the speed definition we will be able to write

$$OA \ . \ \frac{\sqrt{3}}{2\pi} \ v \approx 2^2 \ cm^2/s \ . \tag{13}$$

It is interesting to calculate the value of the diameter *OA*, when the knot is moving with maximum speed experimentally observed: $v \rightarrow c = 2.9979245 \times 10^{10}$ *cm/s* .

From (13) it results:

$$OA \approx \frac{8\pi}{\sqrt{3}c} = 4.8401465 \times 10^{-10} cm. \tag{14}$$

This value is equal to the double of the Compton wavelength associated to the electron $2\lambda_C = h/m_e c \ (1-\cos\pi) = 4.8526204 \times 10^{-10} cm.$, (between the limits of an error of *0,2%*) and with the value of the elementary electric charge $e = 4.8032042 \times 10^{-10} Fr.$ (between the limits of an error of *0,8%*).

Making the hypothesis that the elementary electric charge is built of the sum of the length of the two chords (that torsion the fiber creating the electric field) we can understand the difference between the positive and the negative electric charge (Fig.4a and 4b), considering a movement sense and images - topological inversed - of the two

Δt = 1s ***Not to scale***

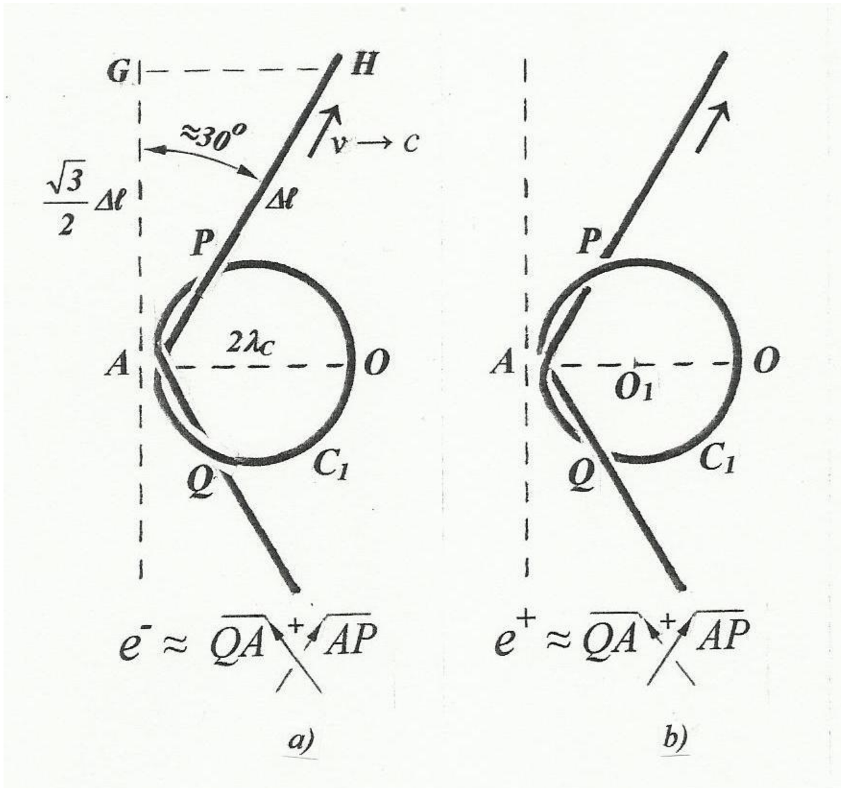


Fig.4 Transformation by inversion of a fundamental field line in a elastic overhand knot (open trefoil knot) with minimum energy. The length of AO segment determined the scale of sketch .

Δt = 1s ***Not to scale***

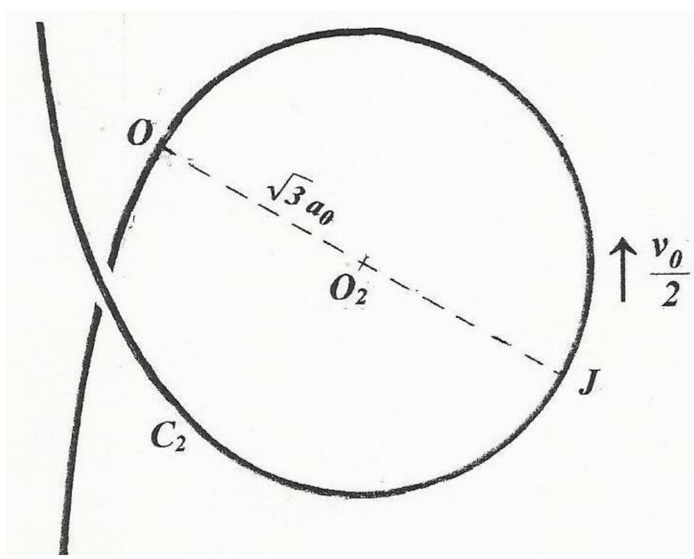


Fig.5 Transformation by inversion of a fundamental field line in a elastic loop which moving with $v_0/2$ velocity. The length of OJ segment determined the scale of sketch .

knots. This interpretation presuppose transition from Gaussian units *(LTM)* to geometric unit *(L)*.In geometrical system we have: $[OA] = L; [AP] = L; [v] = L^{-1}; [c] = L^{-1}; [K] = 1; [e] = L$.

Examining the relation (14) we will conclude that the diameter of the knot and implicit the sum of cords length (the electric charge) are invariant, being expressed depending on geometric constant and the fundamental constant c.

Another interesting application of the transformation by inversion is the interpretation of the product $a_0 v_0$, which intervene in the angular momentum relation $\hbar$ ($\hbar = m_e a_0 v_0$).

The simple examination of the value:

$$a_0 = 0.5291772 \times 10^{-8} \text{ cm.},$$
$$v_0 = 2.1876912 \times 10^{8} \text{ cm/s},$$

leads us to the hypothesis that we have an inversion, as for the values c and $2\lambda_C$.

Making the product $a_0 v_0$, results:

$$a_0 v_0 = 1.1576763 \approx \frac{2}{\sqrt{3}} \text{ cm}^2/\text{s} . \qquad (15)$$

The error made by this approximation is of *0,2%*.

Considering $\sqrt{3}\, a_0$ as a loop diameter (Fig. 5) which generates a rotating body (electron cloud in the form of an apple, approximated spherical - ground state) and $v_0/2$ the speed of the loop, get equation (in geometrical system):

$$\sqrt{3}\, a_0 \,.\, \frac{v_0}{2} \approx 1^2 , \qquad (16)$$

in order to be treated as a transformation by inversion. We are, of course, in case II, case in which, choosing for the smallest measurement unit of *1 cm.*, we have:

$$OJ \approx \sqrt{3}\, a_0 = 1 \text{ cm} , \quad .$$

$$v_0/2 \approx 1 \text{ cm}^{-1},$$

$$K \approx 1^2$$

The two circles (C_1 and C_2) can be reunited in a stable construction (Fig.6), in which over a general movement of slipping that generates the circle C_1 overlaps in the other sense, a rolling that generates by inversion the circle C_2. To remark that $v_0/2$ cannot have the same sense with c , this being the limit speed. Because $v_0 /2 << c$, it results that the radius of the C_2 circle is bigger than the radius of the C_1 circle.

In geometrical system of units:

$QA + AP \approx |e^{\pm}| \approx 4.8\times10^{-10}$cm.

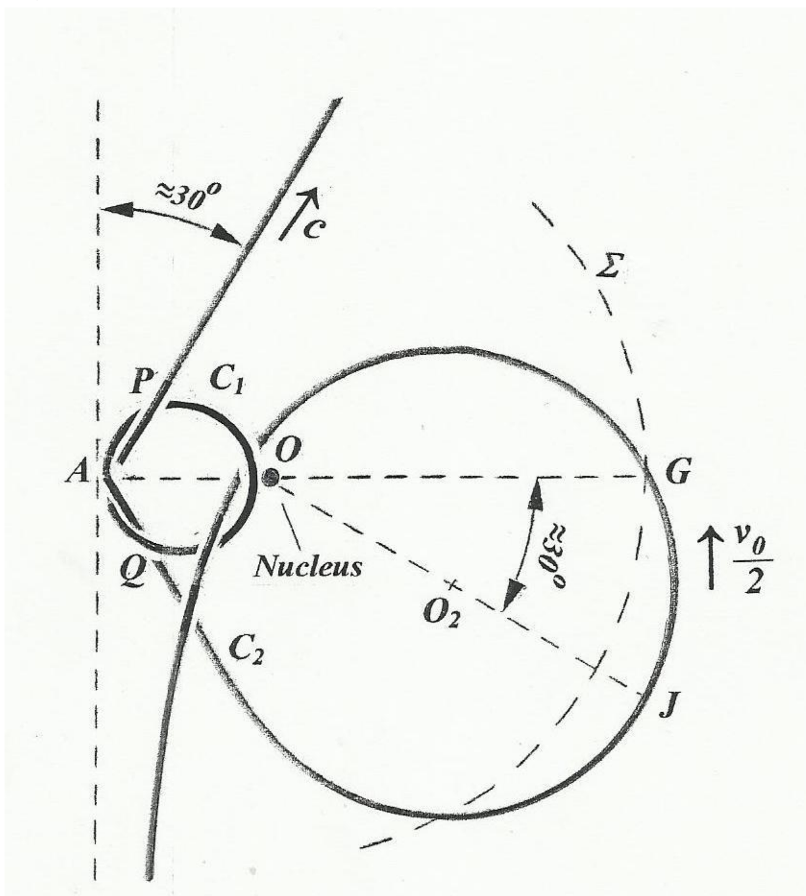
$AO \approx 2\lambda_C \approx 4.8\times10^{-10}$cm.

$OG \approx \frac{3}{2}a_0 \approx 0.8\times10^{-8}$cm.

$OJ \approx \sqrt{3}a_0 \approx 0.9\times10^{-8}$cm.

$v_0/2 \approx 1.1\times10^{8}cm^{-1}$.

$c \approx 3\times10^{10}cm^{-1}$. ***$\Delta t$ = 1s***



Not to scale

Fig.6 The average electron in ground state like a selfconsistence construction joining C_1 knot and C_2 loop. The sphere Σ represents the electron cloud with average radius, $(3/2)a_0$.

Concluding, we will underline that these examples offer an interesting joining between the mathematical geometry and the physical one, that beyond hypothesis can lead to the generalization of the results.

Acknowledgement

I am greatly thankful to professors Nicolae Mihaileanu and Leon Livovschi from Bucharest University for their valuable support.

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Note

Some excerpts from author works complete on understanding about electron :

Table 1

Fundamental physical constants of electron. Comparison between SI and Gaussian units (2012)

The starting point of this table is CODATA recommended Values of the Fundamental Physical Constants-2006, by Peter J. Mohr, Barry N. Taylor, and David B. Newell.

Opportunity of used Gaussian units shown in [1-3]. An amendment to Bohr magneton measure unit was developed in [3]. This amendment consist in transfer of 1/c factor of units in torque relation imply the change of unit emu → esu. Importance of double Compton wavelength result in description of symmetry phenomena [4].

Quantity, symbol	SI units	Gaussian units
speed of light in vacuum (c, c_0)	2.997 924 58 $\times 10^{8}$ m s^{-1}	2.997 924 58 $\times 10^{10}$ cm s^{-1}
magnetic constant (vacuum permeability), (μ_0)	12.566 370 614 . . . $\times 10^{-7}$ N A^{-2}	1
electric constant (vacuum permittivity), (ε_0)	8.854 187 817 . . . $\times 10^{-12}$ F m^{-1}	1
characteristic impedance of vacuum (Z_0)	376.730 313 461 . . . Ω	1
Compton wavelength (λ_C)	2.426 310 2175 (33) $\times 10^{-12}$ m	2.426 310 2175 (33) $\times 10^{-10}$ cm
electron g-factor (g)	-2.002 319 304 3622(15)	-2.002 319 304 3622(15)
Bohr magneton (μ_B)	927.400 915(23) $\times 10^{-26}$ JT^{-1}	2.780 277 998(69) $\times 10^{-10}$ esu
electron magnetic moment (μ_e)	-928.476 377(23) $\times 10^{-26}$ JT^{-1}	-2.783 502 153(69) $\times 10^{-10}$ esu
spin magnetic moment [$\mu_S = g\,(e/2m_e)\mathbf{S}$]	1.608 168 258(40) $\times 10^{-23}$ JT^{-1}	4.821 167 12(12) $\times 10^{-10}$ esu
electron charge (e)	1.602 176 487(40) $\times 10^{-19}$ C	4.803 204 27(12) $\times 10^{-10}$ esu
double Compton wavelength ($2\lambda_C$)	4.852 620 4350(67) $\times 10^{-12}$ m	4.852 620 4350(67) $\times 10^{-10}$ cm
fine-structure constant (α)	7.297 352 5376 (50) $\times 10^{-3}$	7.297 352 5376 (50) $\times 10^{-3}$
inverse fine-structure constant (α^{-1})	137.035 999 679(94)	137.035 999 679(94)
Bohr radius (a_0)	0.529 177 208 59(36) $\times 10^{-10}$ m	0.529 177 208 59(36) $\times 10^{-8}$ cm
Bohr speed (αc, v_0)	2.187 691 254 1(14) $\times 10^{6}$ m s^{-1}	2.187 691 254 1(14) $\times 10^{8}$ cm s^{-1}
average radius of electron cloud (1s), $\langle r_1 \rangle = 3a_0/2$	0.793 765 812 88(53) $\times 10^{-10}$ m	0.793 765 812 88(53) $\times 10^{-8}$ cm
Planck constant (h)	6.626 068 96(33) $\times 10^{-34}$ J s	6.626 068 96(33) $\times 10^{-27}$ erg s
Planck constant, reduced ($\hbar$)	1.054 571 628(53) $\times 10^{-34}$ J s	1.054 571 628(53) $\times 10^{-27}$ erg s
spin angular momentum ($\mathbf{S} = \sqrt{3}\hbar/2$)	9.132 858 19(45) $\times 10^{-35}$ J s	9.132 858 19(45) $\times 10^{-28}$ erg s
electron rest mass (m_e)	9.109 382 15(45) $\times 10^{-31}$ kg	9.109 382 15(45) $\times 10^{-28}$ g
Newtonian constant of gravitation (G)	6.674 28(67) $\times 10^{-11}$ $m^3 kg^{-1} s^{-2}$	6.674 28(67) $\times 10^{-8}$ $cm^3 g^{-1} s^{-2}$
classical electron radius (r_e, r_0)	2.817 940 2894 (58) $\times 10^{-15}$ m	2.817 940 2894 (58) $\times 10^{-13}$ cm
Thomson cross section (σ_e)	0.665 245 8558 (27) $\times 10^{-28}$ m^2	0.665 245 8558 (27) $\times 10^{-24}$ cm^2
Rydberg constant (R_∞)	10 973 731.568 527(73) m^{-1}	1.097 373 156 852 7(73) $\times 10^{5}$ cm^{-1}

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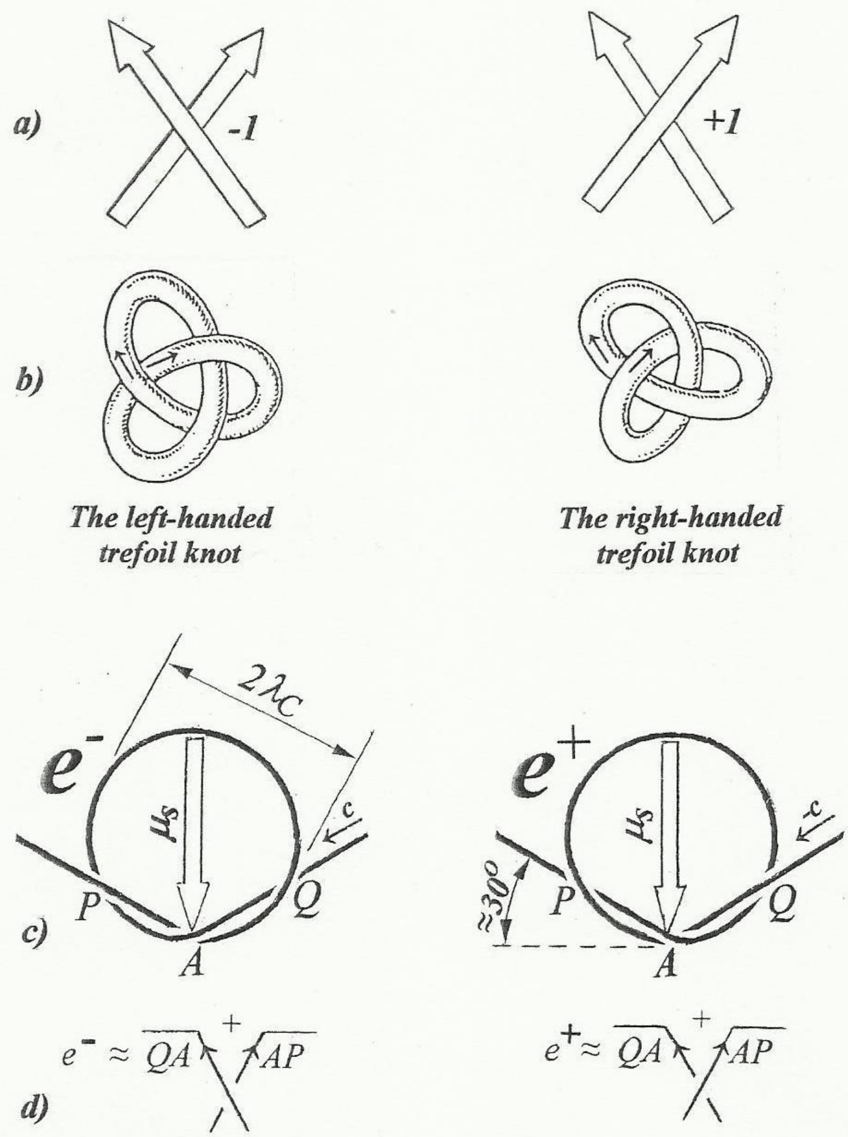
Elementary charge and crossing sign



a) Convention on the crossing sign (P.G. Tait, K. Reidemeister et al.).
b) Sacharov's interpretation in connection with trefoil knot, as a "base" of electric charge.
c) Interpretation of elementary charge (as the sum of two stringh that intersect, of a elastic overhand knot with minimum energy, on the fundamental field line, according to the equalities $|e^{\pm}| \approx |\mu_s| \approx 2\lambda_C \approx 4.8\times10^{-10}$ cm, in a geometric, one-dimensional system by transition LTM Gaussian $\rightarrow$L).
d) Symbolic notation proposed for elementary charge.

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Fractional electric charges

$$e^{+} = \frac{6}{6}\,(\overline{QA}\overset{+}{\times}\overline{AP}) = +e$$

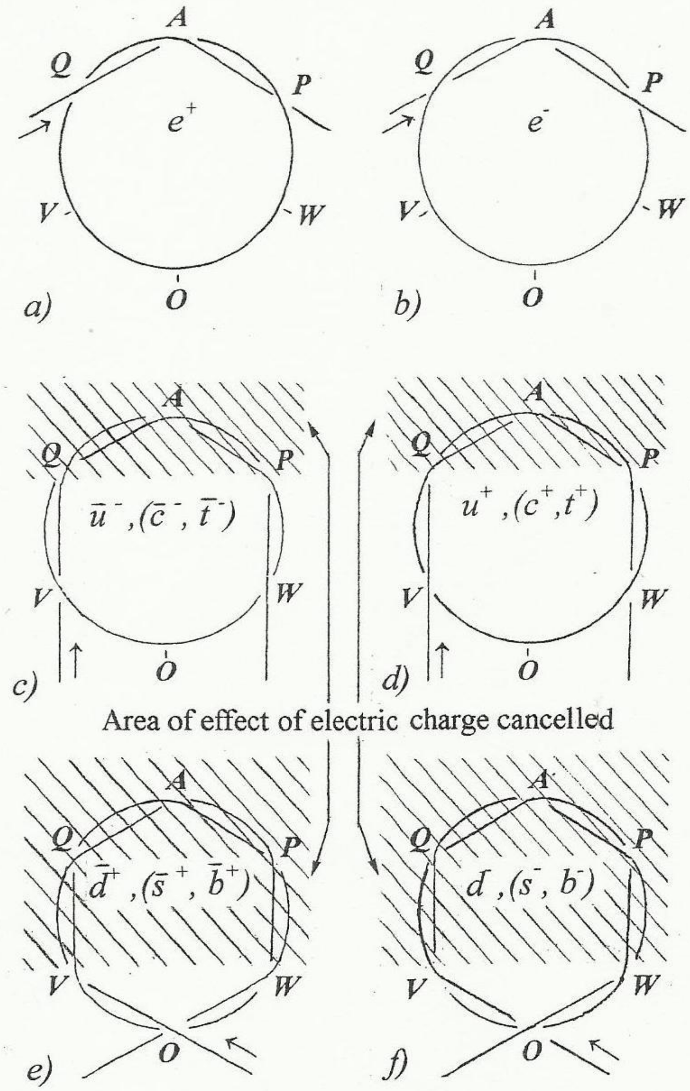
$$e^{-} = \frac{6}{6}\,(\overline{QA}\overset{+}{\times}\overline{AP}) = -e$$

$$u^{+} = \frac{4}{6}\,(\overline{VQ}\overset{+}{\times}\overline{PW}) = +\frac{2}{3}e$$

$$\overline{u}^{-} = \frac{4}{6}\,(\overline{VQ}\overset{+}{\times}\overline{PW}) = -\frac{2}{3}e$$

$$d^{-} = \frac{2}{6}\,(\overline{OV}\overset{+}{\times}\overline{WO}) = -\frac{1}{3}e$$

$$\overline{d}^{+} = \frac{2}{6}\,(\overline{OV}\overset{+}{\times}\overline{WO}) = +\frac{1}{3}e$$



Fractional electric charges, which enters the structure of quarks, presented in comparison with the electric charge of the electron (positron): Starting from elementary electric charge of electron, we can imagine fractional electric charges as parts of complex knots. Considering "knot energy" directly proportional to the length of the arc of a circle which determines the electric charge effect and the circle divided into six sectors, we have,

b) Full electric charge (of electron, e^{-}) = arc AQVOWPA/6 = -(6/6)e = -e;

d) Fractional electric charge of quarks u, c, t = arc QVOWP/6 = +(4/6)e = +(2/3)e;

f) Fractional electric charge of quarks d, e, f = arc VOW/6 = -(2/6)e = -(1/3)e.

Electrical charges of the positron and anti-quarks are shown in images a), c), e).

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Spin motion and 4π rotation

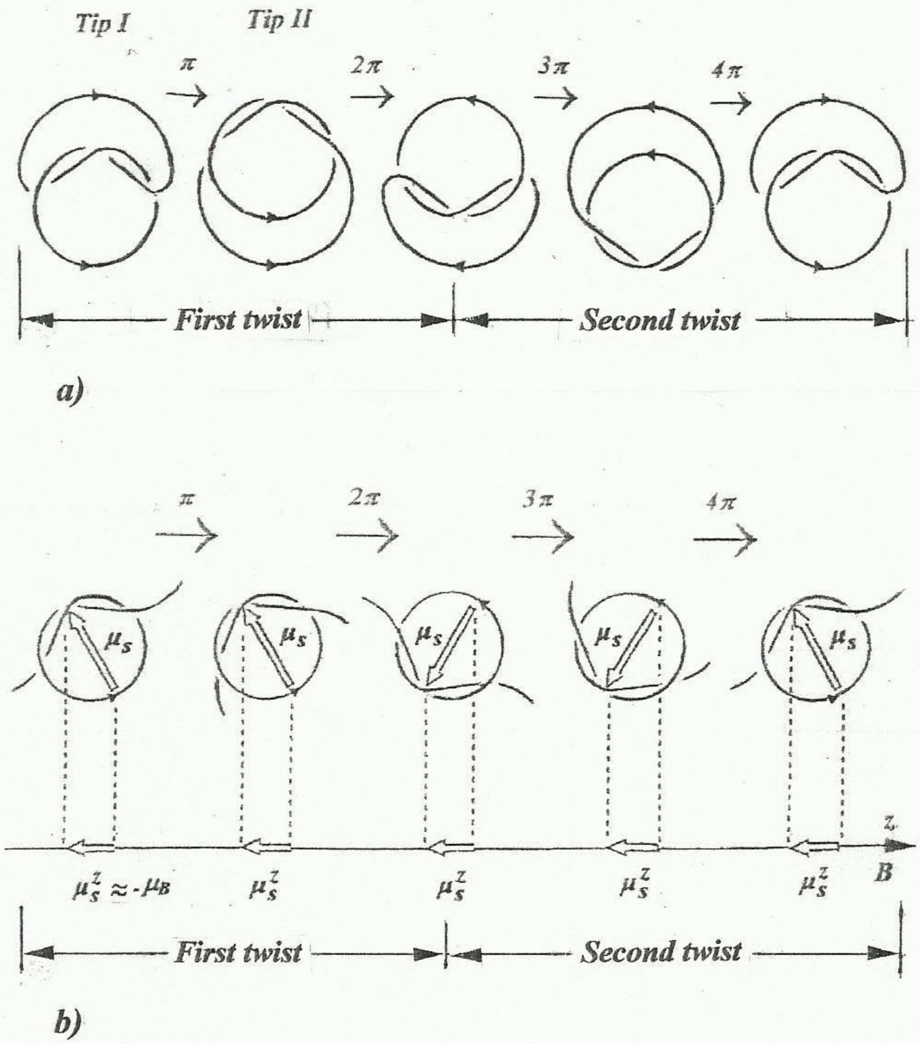

Illustration of the radial spin motion, as an internal double twist, starting from trefoil knot:

a) The spin motion with 4π internal rotations in the case of trefoil knot.

b) The spin motion with 4π rotations in the case of elastic overhand knot (open trefoil knot) identified with extended quantum electron (charge). Vector μ_s projection on the $0z$ axis (rotational component) is permanently $\mu_s^z \approx -\mu_B$.

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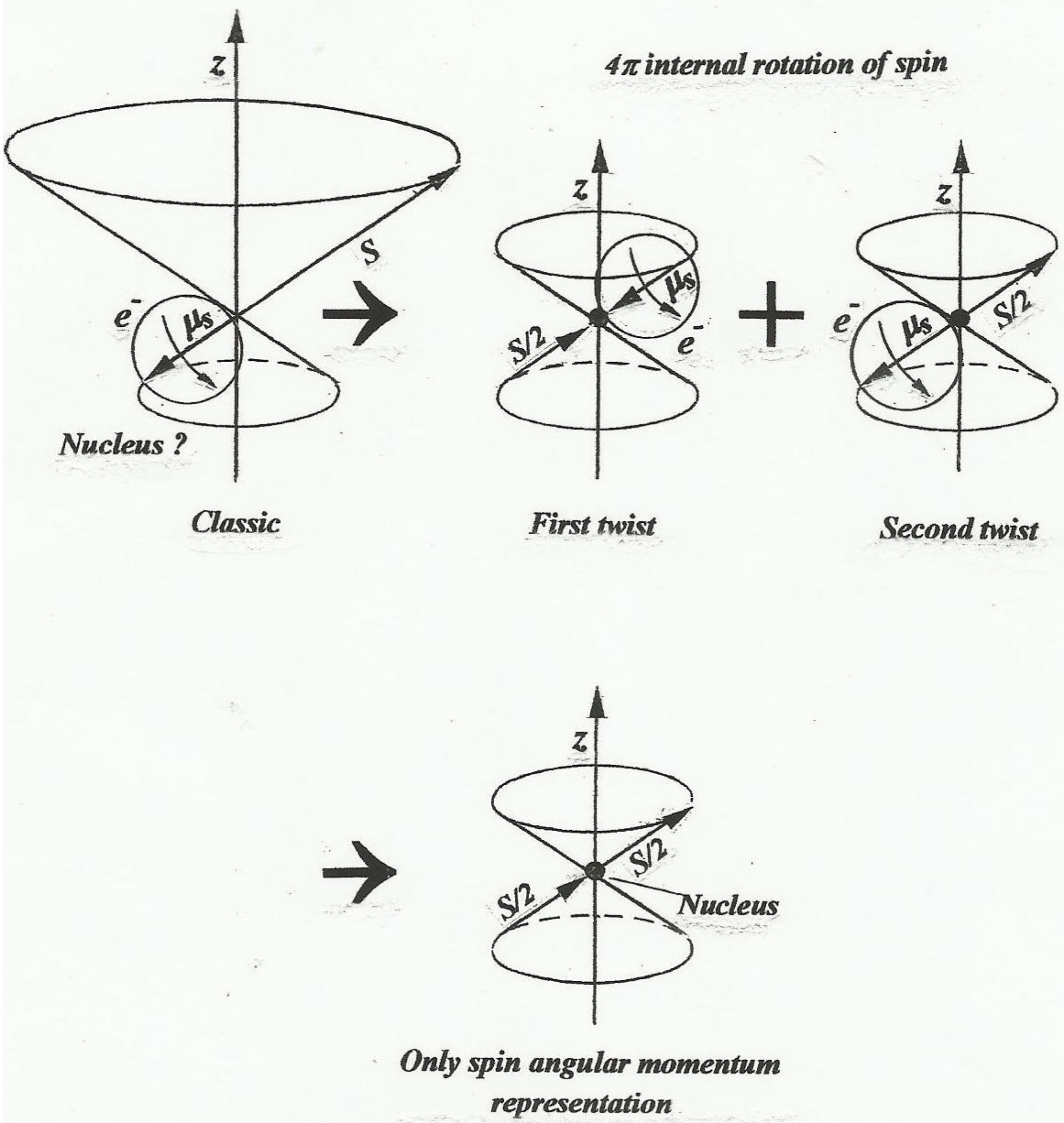
Interpretation of classical spin image



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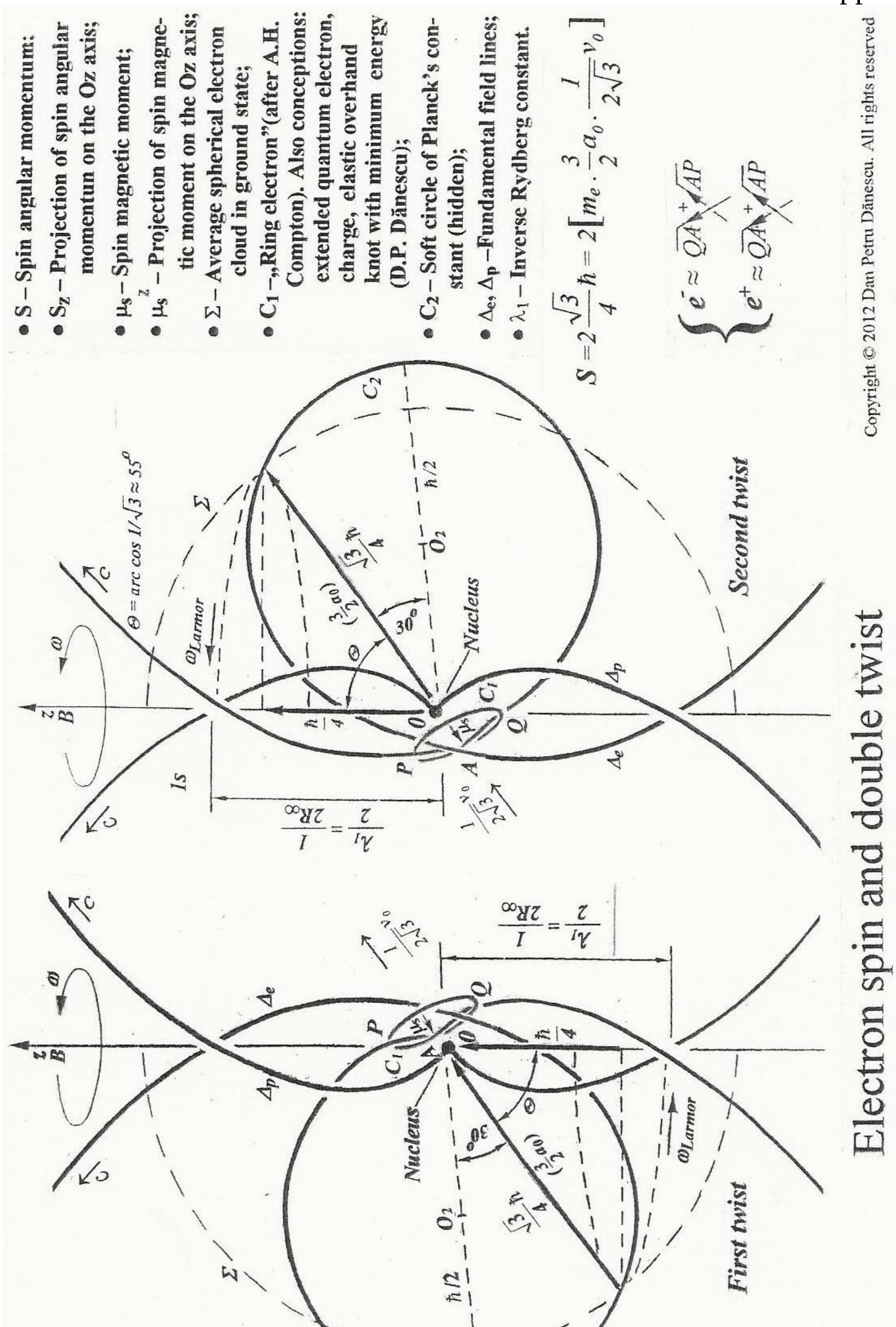
Electron spin and double twist
• S – Spin angular momentum;
• S_z – Projection of spin angular momentum on the Oz axis;
• μ_s – Spin magnetic moment;
• μ_s^z – Projection of spin magnetic moment on the Oz axis;
• Σ – Average spherical electron cloud in ground state;
• C_1 – „Ring electron"(after A.H. Compton). Also conceptions: extended quantum electron, charge, elastic overhand knot with minimum energy (D.P. Dănescu);
• C_2 – Soft circle of Planck's constant (hidden);
• Δ_e, Δ_p – Fundamental field lines;
• λ_1 – Inverse Rydberg constant.
$S = 2\frac{\sqrt{3}}{4}\hbar = 2\left[m_e \cdot \frac{3}{2}a_0 \cdot \frac{1}{2\sqrt{3}}v_0\right]$
$e^- \approx \overline{QA} + \overline{AP}$
$e^+ \approx \overline{QA} + \overline{AP}$
$\Theta = arc\ cos\ 1/\sqrt{3} \approx 55^o$
First twist
Second twist
Nucleus
ω_{Larmor}
$\frac{\sqrt{3}}{4}\hbar$
$\hbar/2$
$\frac{\hbar}{4}$
30^o
$(\frac{3}{2}a_0)$
$\lambda_1 = \frac{1}{2R_\infty}$
$\frac{1}{2\sqrt{3}}v_0$
1s
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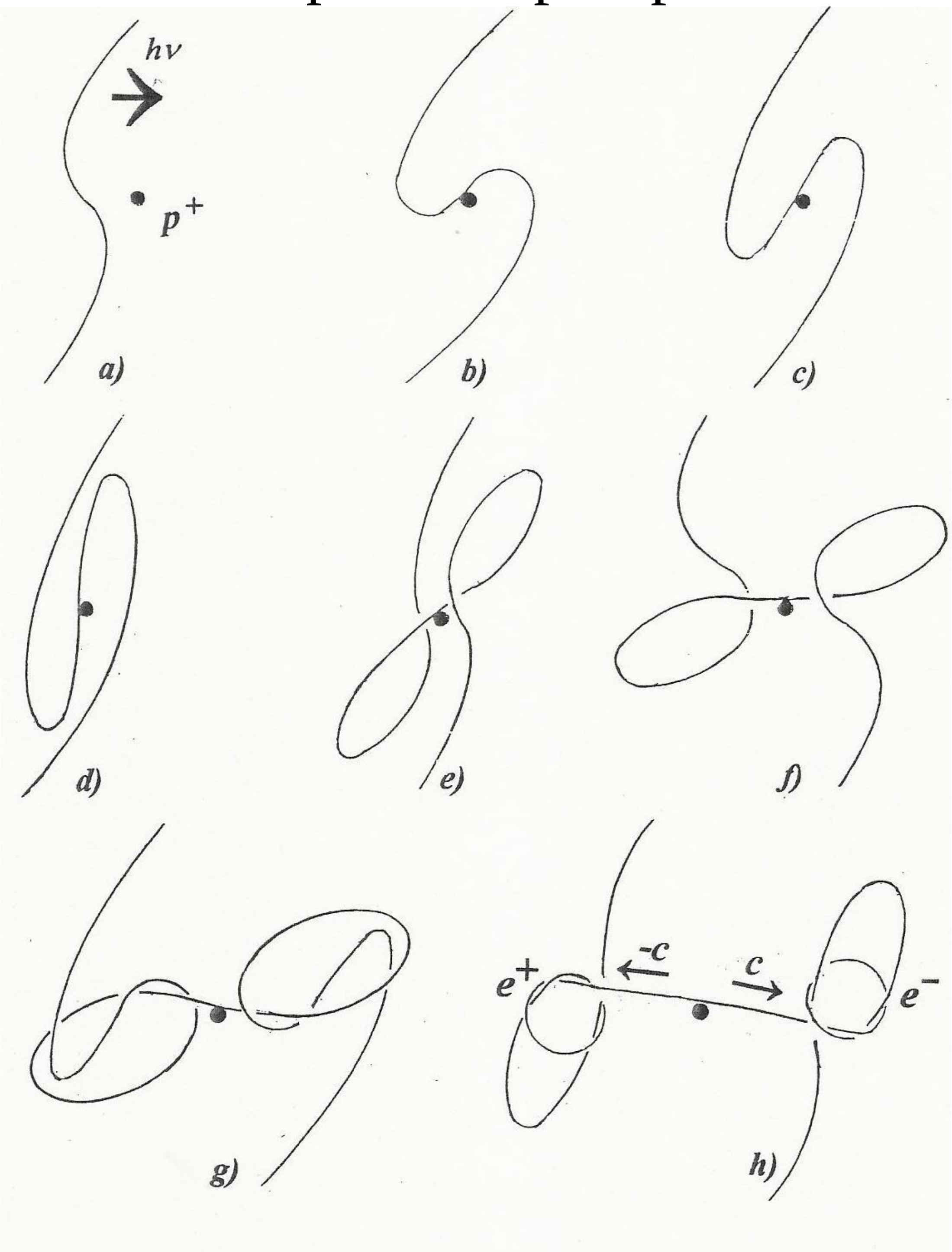
Electron-positron pair production



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